Applied Analysis and Mathematical Physics II

This session brings together distinguished researchers in Applied Analysis working on models from Mathematical Physics. The aim is to show some of the latest advances in this area, including topics on quantum mechanics, kinetic theory and fluid dynamics.

Organizadora: Marina A. Ferreira, Centre National de la Recherche Scientifique (CNRS) & Université de Toulouse, *marina.ferreira@math.univ-toulouse.fr*

Orador 1

Dmitry Vorotnikov, Centro de Matemática da Universidade de Coimbra,
mitvorot@mat.uc.pt

Title: Weighted ballistic minimization problems and Dafermos' principle
Abstract: We study the systems of nonlinear PDEs that can be written in the form

$$\partial_t v = L(\mathbf{F}(v))$$

with some symmetry assumptions that formally yield conservation of an entropy. Here

$$\mathbf{F} : \mathbb{R}^n \to \mathbb{R}_+^{n\times n}$$

is a convex function in the sense of Löwner, and L is a linear operator. These problems can be viewed as generalized conservation laws. The examples include the equations of motion of compressible barotropic fluids, the NLS equations, various models with quadratic nonlinearity such as Euler's equations of inviscid fluids, the ideal MHD and many others. We discuss the dual (ballistic) formulation of such problems in the spaces of matrix-valued measures. Among other results, we prove that no subsolution of such problems can dissipate the entropy faster than the strong solution provided that the latter exists.

Orador 2

Ana Jacinta Soares, Centro de Matemática da Universidade do Minho,
ajsoares@math.uminho.pt

Title: On the Boltzmann equation modelling multi-component reactive system
Abstract: The kinetic theory of gases provides a powerful framework to describe the time evolution of a rarefied gas whose particles move randomly and undergo binary

collisions according to an interaction potential. The central model in this theory is the Boltzmann equation (BE), which constitutes an integro-partial differential equation with integral terms describing collisions and other types of interactions among particles. Specifically, for a mixture undergoing chemical reactions, many mathematical complexities arise, mainly due to the rearrangement of masses and redistribution of energies. The BE becomes too complex for mathematical treatment, practical computations, and numerical simulations due to the lack of many symmetry properties and the highly nonlinear character of the integral terms.

To address this difficulty, simplified collisional models can be employed. The Bhatnagar-Gross-Krook (BGK) model replaces the integral collision term in the BE with a single relaxation term that drives the evolution towards a local Maxwellian equilibrium at an exponential rate. When dealing with more complex systems, a single relaxation term may not adequately capture the diverse processes involved in the dynamics, and multiple relaxation terms should be considered.

In this talk, we will present a BGK-type model with multiple relaxation terms, designed to better represent the various mechanisms involved in the system and, at the same time, to satisfy good mathematical properties.

Orador 3

Jean-Baptiste Casteras, CMAFcIO, Universidade de Lisboa,
jeanbaptiste.casteras@gmail.com

Title: Almost sure existence of solutions for the cubic Schrödinger equation

Abstract: In this talk, we will discuss the existence of solutions for the cubic Schrödinger type equation (NLS) on the whole space with rough initial data. Although such a problem is known to be ill-posed, we show that a randomisation of the initial data yields almost sure local well-posedness. Using estimates in directional spaces, we improve and extend known results for the standard Schrödinger equation in various directions: higher dimensions, more general operators, weaker regularity assumptions on the initial conditions. In particular, we show that in 3D, the classical cubic NLS is stochastically, locally well-posed for any initial data with regularity in H^ε for any $\varepsilon > 0$, compared to the known results $\varepsilon > \frac{1}{6}$.