Álgebra, Grupos e Combinatória

Organizer: Oscar Morales - Mathematics Center of the University of Porto

Descrição da Proposta de sessão paralela: A sessão de Álgebra, Grupos e Combinatória tem como objetivo principal explorar as interconexões entre tais disciplinas, bem como as diversas técnicas utilizadas em cada uma delas, mesmo as que nem sempre têm interseção aparente. Porém, áreas como Representações de Álgebras de Lie e de Álgebras de Vértice sobrepoem-se e criam a necessidade e de formar grupos de investigação multidisciplinares que permitam resolver os problemas nesta área da física-matemática. Além disso, com esta sessão pretende-se contribuir para o fortalecimento da interação entre investigadores de Portugal e do Norte de Espanha.

Speaker 1 - Xabier García Martínez, University of Vigo.

Title: A characterisation of Lie algebras using ideals and subalgebras.

Abstract: Inspired by the categorical characterisation of Lie algebras amongst all varieties of non-associative algebras using *algebraic exponents* [1], the following result was proved in [2]:

Theorem 1. *Suppose that $\mathfrak{M}$ is a non-trivial variety of non-associative algebras over a field of zero characteristic satisfying the following two conditions:*

- *every subalgebra of every free algebra is free*
- *for every ideal I in every algebra, I^2 is also an ideal*

Then $\mathfrak{M}$ is the variety of Lie algebras.

A variety satisfying the first condition is called *Nielsen-Schreier*, whilst a variety satisfying the second condition is said to be a 2-*variety*.

In this talk we will first focus on giving a wide and intuitionistic idea about this result, together with its motivation and origins. Then we will explain the methods used to prove it, which include homological and computational algebra, together with Gröbner bases for operads.

Joint work with Vladimir Dotsenko (Université de Strasbourg)

Bibliography

[1] V. Dotsenko and X. García-Martínez. A characterisation of Lie algebras using ideals and subalgebras To appear in *Bull. Lond. Math. Soc.* (2024).

[2] X. García-Martínez and T. Van der Linden. A characterisation of Lie algebras via algebraic exponentiation. *Adv. Math.* 341, (2019), 92–117.

Speaker 2 - André Carvalho, University of Porto.

Title: On some generalizations of the conugacy problem.

Abstract: The conjugacy problem is one of the three fundamental algorithmic problems introduced by Dehn at the beginning of the twentieth century. Since then, it has been studied from different viewpoints across various classes of groups. In this talk, we will explore some generalizations of the conjugacy problem and discuss a language-theoretic approach to solving them in (virtually) free groups.

Speaker 3 - Inês Legatheaux Martins, University of Lisbon.

Title: On some rook diagram algebras and Schur-Weyl dualities via Schur algebras

Abstract: Schur-Weyl duality between the symmetric group S_n and the general linear group $GL_d(\mathbb{C})$ on tensor spaces is a centrepiece of representation theory with far-reaching consequences. Throughout the years, various instances of this phenomenon have been observed for subgroups of $GL_d(\mathbb{C})$ and other diagram algebras (that is, algebras with a basis consisting of diagrams). For instance, the orthogonal group is in Schur-Weyl duality with the Brauer algebras and the Weyl group is related in this way to the partition algebras. In the past 40 years, it has been established that some of these Schur-Weyl dualities remain true if one replaces $\mathbb{C}$ by an infinite field $\mathbb{F}$. This is largely due to Schur algebras. Introduced by J. A. Green in 1980, these finite-dimensional algebras and their extensions provide a natural setting for studying the (modular) representation theories of diagram algebras. In 2002, for a field $\mathbb{F}$ of characteristic zero, L. Solomon proved a Schur-Weyl duality between $GL_($F and the rook monoid R_n on tensor spaces. His result led to the investigation of other interesting diagram algebras, such as rook partition algebras, rook Brauer algebras and their planar versions. The aim of this talk is to define Schur algebras analogues that are in Schur-Weyl duality with the aforementioned (rook) diagram algebras. We intend to give an overview of previously known and new results that make use of this approach.